T-Burner Method of Determining the Acoustic Admittance of Burning Propellants

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T-burner experiments have provided the most fruitful technique for obtaining the acoustic admittance of burning solid propellants The objections to the findings of these experiments are: 1) the admittance values are questioned on the grounds that acoustic losses in the apparatus are not well enough understood for adequate corrections to be made, and 2) relative acoustic losses are great enough that some operational propellants, known to produce oscillations in motors, burn stably in the T-burner The first objection is largely removed by demonstrating that three different methods for operating the burner and analyzing the data give results in close agreement Besides confirming the results of the other two, one method described here also provides a means to study more stable propellants, thus removing the second objection to the T-burner technique

Nomenclature

A_n = amplitude of acoustic displacement
a_n = complex constant
c = velocity of sound
j = square root of minus one
k_n = complex constant
l = length of gas column
$\bar{p}$ = mean pressure
p = acoustic pressure
r = linear burning rate
t = time
$\bar{v}$ = mean velocity
v = acoustic velocity
W = complex quantity equal to $\alpha + j\omega$
x = position coordinate
Y = specific acoustic admittance
y = reduced specific acoustic admittance
α = exponential growth constant
β = dissipation function
γ = ratio of specific heats
Ψ = characteristic spatial function
ζ = acoustic displacement
ρ = density of gas
ω = angular frequency

Subscripts

b = burning propellant surface
d = damping
0 = value at $x = 0$
l = value at $x = l$
s = propellant

Introduction

DURING the past three or four years, the T-burner has been rather extensively used to study experimentally acoustic combustion instability of burning solid propellants The distinguishing feature of this burner is the location of the nozzle or vent in the tubular side-wall midway between the ends, where the propellant is located (Fig 1) The virtue of the T-burner is its ability to yield reproducible quantitative experimental data from which the acoustic admittance of burning propellants can be calculated Prior to the development of the T-burner, there was no reliable data that could be compared directly with theory

Even though quantitative data can now be obtained, certain unverified assumptions must be made in the acoustic admittance calculations The purpose of this paper is to present evidence that seems to dispel some of the doubts about these assumptions The evidence is in the form of agreement between the admittance values obtained for the same propellant by three different variations of the T-burner method

After a simplified theory for the T-burner experiment is developed, the basic technique described previously by Horton[1] will be discussed Two substantially different methods will then be desc\iibed, one of which may extend the use of the burner to more stable-burning propellants Data obtained by all three methods will then be compa\ied

Presented as Preprint 64-137 at the Acoustic Combustion Instability session (cosponsored by the Department of Defense Technical Panel on Solid Propellant Instability of Combustion) at the AIAA Solid Propellant Rocket Conference, Palo Alto, Calif, January 29–31, 1964; revision received March 16, 1964

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Theory

It has been found that when end-burning grains are used in the T-burner, only longitudinal acoustic modes are excited This fact, together with the observation that the oscillations are observed to increase in amplitude exponentially with time during the growth period, somewhat simplifies the theoretical analysis It is known beforehand that the equation that describes the observed gas particle displacement ζ can be

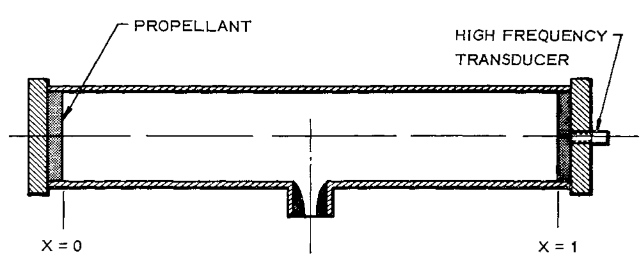

Fig 1 Configuration of T-burner; the nominal diameter of burners used in work reported here is 1 5 in

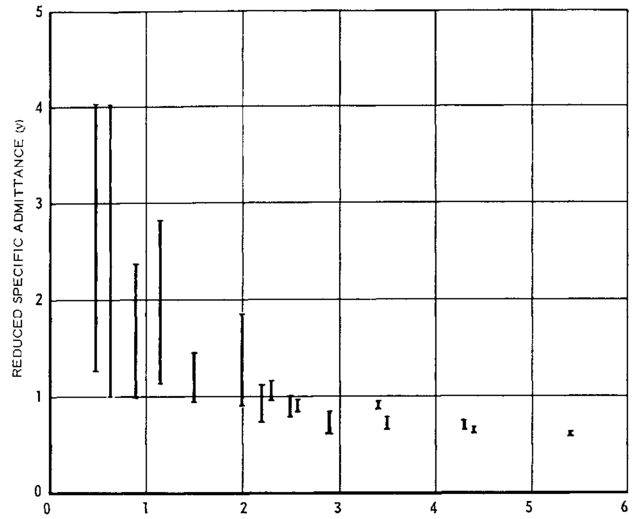

Fig 2 Specific admittance data obtained at Naval Ordnance Test Station using method I

written

$$\zeta = \sum_{n=1} A_n \Psi(x) e^{(\alpha_n + j\omega_n)t} \tag{1}$$

The problem at hand is to find how the measurable quantities α_n and ω_n are related to more fundamental properties of the resonating system, in particular, those of the burning propellant surface

The problem will be attacked by writing an equation describing damped wave motion of a differential element of propellant gases:

$$\frac{\partial^2 \zeta}{\partial t^2} + \beta \frac{\partial \zeta}{\partial t} = c^2 \frac{\partial^2 \zeta}{\partial x^2} \tag{2}$$

where β is a frequency-dependent "dissipation function" that describes the particular processes that are occurring to degrade wave energy A solution is sought for the time dependence of particle displacement ζ in terms of the acoustic admittances of the gas boundaries at $x = 0$ and $x = 1$ (Fig 1) It should be pointed out that the omission of any consideration of mean flow effects in this analysis is intentional As discussed later, these effects can be treated as part of the effective end admittances It will be assumed that the function describing the dependence of the displacement on position $\Psi(x)$ can be written as

$$\Psi(x) = \sin k_n(x - a_n) \tag{3}$$

where k_n and a_n are complex constants to be evaluated from the boundary conditions

It will be convenient to write $\alpha_n + j\omega_n = W_n$ By combining Eqs (1) and (3), and substituting into (2), one finds that

$$W_n = k_n c/(1 - j\beta_n/W_n)^{1/2} \tag{4}$$

An attempt will now be made to evaluate k_n in terms of the effective specific acoustic admittances of the gas boundaries Y_0 and Y_l Since

$$Y = \pm \frac{v}{p} = \pm \frac{\partial\zeta/\partial t}{\rho c^2(\partial\zeta/\partial x)}$$

where p and v are the acoustic pressure and velocity and ρ is the gas density, it follows that

$$\tan(-k_n a\ ) = k_n c(\rho c Y_0/jW_n)$$
$$\tan(l - a_n) = -k_n c(\rho c Y_l/jW_n) \tag{5}$$

These equations can be combined to obtain

$$\tan k_n l = -k_n c[(\rho c Y_0/jW_n) + (\rho c Y_l/jW_n) + k_n c(\rho c/W_n)^2 Y_0 Y_l \tan k_n l] \tag{6}$$

Realizing that Y_0 and Y_l are, in practice, small quantities, one may approximate (6) to the first order by

$$k_n \simeq \frac{n\pi}{l}\left[1 + j\left(\frac{c}{l}\right)\left(\frac{\rho_c Y_0}{W_n} + \frac{\rho c Y_l}{W_n}\right)\right] \tag{7}$$

Equations (4) and (7) can now be combined to yield the equation

$$W_n \simeq \left(\frac{\pi n c}{l}\right)\frac{\{1 + j(c/l)[(\rho c Y_0/W_n) + (\rho c Y_l/W_n)]\}}{[1 - j\beta\ /W_n]^{1/2}} \tag{8}$$

which relates W_n, Y_0, Y_l, l, ρ, and c Further approximation will be useful to obtain a more explicit equation for W_n This is accomplished by the rather bold step of evaluating the coefficients in (8) containing Y_0, Y_l, and β_n, with W set equal to $n\pi c/l$ This step is justified in practice because these quantities are found to be small in comparison to W_n Thus, the first-order approximation of W_n can be written as

$$W_n \simeq \frac{n\pi c}{l} + j\left[\frac{\beta_n}{2} + \frac{c}{l}\ (\rho c Y_0 + \rho c Y_l)\right] \tag{9}$$

or, in terms of its complex components,

$$\alpha_n \simeq -[(\beta_n/2) + (c/l)R\ (\rho c Y_0 + \rho c Y_l)] \tag{10}$$

and

$$\omega_n \simeq (c/l)[n\pi - I_m(\rho c Y_0 + \rho c Y_l)] \tag{11}$$

It is seen that the growth constant of the fundamental mode α is made up of two terms, one representing energy transferred through the end boundaries and the other the distributed energy losses, or $\alpha = \alpha_b + \alpha_d$ If Y_0 and Y_l are replaced by Y_b, the effective specific acoustic admittance of the burning propellant surface, the criterion for stability that immediately follows from Eq (10) is

$$Re(Y_b) + (l/4)(\beta/\rho c^2) > 0$$

With this brief theoretical discussion to serve as a common basis, the following describes three different experimental methods for obtaining numerical values of $Re(Y_b)$

Method I

The method that has been used at both the Naval Ordnance Test Station (NOTS) and the Ballistics Research Laboratories to measure acoustic admittances of burning propellants

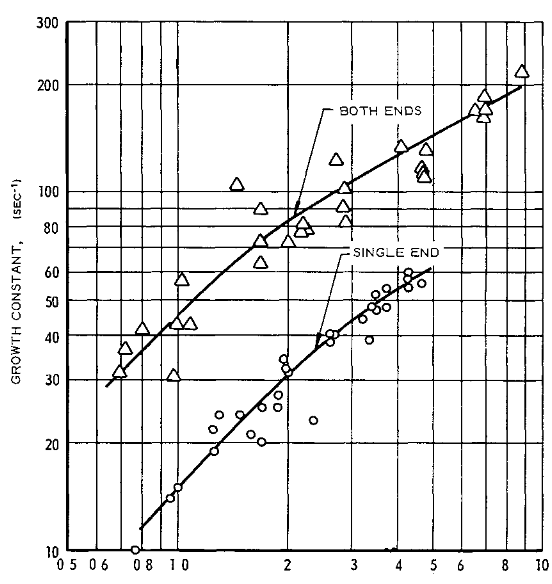

Fig 3 Growth constant data for propellant burned in one or both ends of the T-burner This data was obtained at the University of Utah

has been to vent the burner into a large, pressurized tank and then to measure both the rate of growth of oscillations after ignition of the propellant samples and their rate of decay after the propellant has been consumed [1,2] The premise upon which this method depends is that the rate of acoustic energy dissipation is the same during both periods The decay rate α_d, which exists after burn out when the end admittances become equal to zero, is thus a measure of the rate of energy dissipation during both the growth and decay periods If this is true, then the real part of the specific acoustic admittance of the burning propellant, as given by Eq (10), is related to the growth and decay constants by

$$Re(\rho c Y_b) = -(l/2c)(\alpha - \alpha_d) \quad (12)$$

with $\alpha_d = -\beta/2$

Figure 2 shows data for a composite propellant obtained at NOTS by this method The propellant was a polybutadiene-acrylic acid/ammonium perchlorate formulation with copper chromite catalyst in the proportions of 18/80/2 Figure 2 is a plot of frequency vs the reduced specific acoustic admittance y, which is defined as

$$y = -(\bar{p}/\bar{v})Re(Y_b)$$

or

$$y = -(\bar{p}/r\rho_s c)Re(\rho c Y_b) \quad (13)$$

where $\bar{p}$ and $\bar{v}$ are mean values of the pressure and velocity of the products of combustion, r the linear burning rate, and ρ the density of the unburned propellant For these data, the mean pressure was 200 psi, and the value of the dimensionless group $\bar{p}/r\rho c$ was approximately 106

As lower frequencies are tested with this method, a true decay constant becomes difficult to measure, and eventually the rate of decay of oscillations no longer is exponential with time Effectively, α_d changes during the decay period This behavior is responsible for the rather wide spread in the data, as shown in Fig 2 The upper and lower limits actually represent calculations made using maximum and minimum values of the decay constant

The concern over the validity of this method results naturally from the important assumption that the dissipation rate is the same in both the growth and decay periods Change of state of the combustion gases through loss of energy by conduction to the walls of the burner is being neglected This may have an important effect on the growth and decay rates

Concern has also been expressed over differences in the mean flow fields during growth and decay periods This

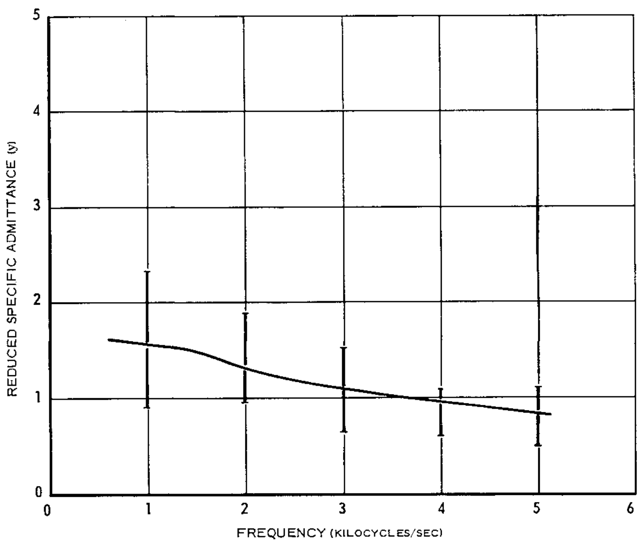

Fig 4 Reduced admittance data calculated from the growth constant data of Fig 3 using method II

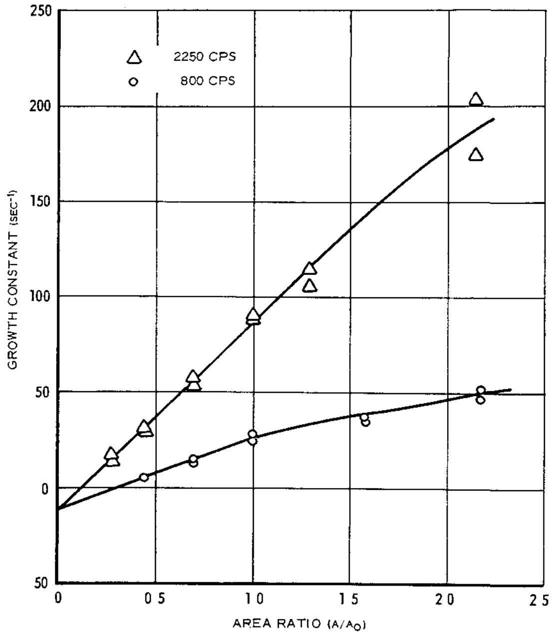

Fig 5 Data showing the effect of changes in the burning surface area on the growth constant (method III)

problem has recently been discussed by McClure et al [3] Their work showed that the influence of the mean flow field can be treated as a decrease in the admittance of burning surface by an amount equal to $\bar{v}/\gamma\bar{p}$, where γ is the ratio of specific heats of the gas Thus, the effective acoustic admittance calculated by this method includes the effects of the mean flow

Some confidence in the decay-rate assumption of this method has been generated by recent work of Horton and McGie,[4] who were able to interpret experimental values of the dissipation function β in terms of the theory of Epstein and Carhart[5] for acoustic energy dissipation by a particle-laden gas

Method II

This method dispenses altogether with decay-rate measurements by employing only measurements of growth constants [6] The technique is to compare data for experiments where propellant is burned in only one end of the burner with those obtained with propellant in both ends The assumption required to calculate admittances from this method is that the distributed losses are the same on both sides of the burner regardless of whether or not propellant is burned at both ends If this assumption is valid, then Eq (10) can be used to show that

$$Re(\rho c Y_b) = (l/c)(\alpha_1 - \alpha_2) \quad (14)$$

where α_2 and α_1 are measured growth constants for propellant in both ends and one end, respectively

Figure 3 shows growth-rate data obtained at the University of Utah or the same propellant mentioned previously Figure 4 shows corresponding values for the reduced specific admittance for this propellant as calculated by method II

The weakness of this method is that the effect of the unsymmetrical gas column is not known Acoustic energy dissipation may be significantly different in the "cold" end of the burner than it is in the end where propellant is burning

Method III

Also dispensing with decay-rate measurements, method III really represents a generalization of the previous approach, since it relies upon the effect that changes in the total burn-

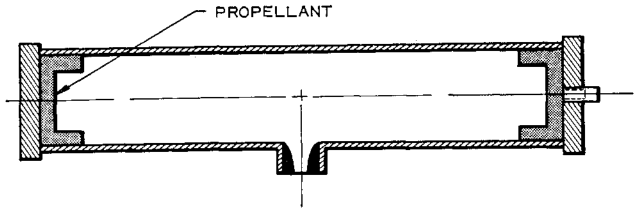

Fig 6 Technique used in method III for increasing the propellant burning surface areas

ing propellant area have on the rate of growth of oscillations, the burner geometry remaining otherwise approximately constant Assume that part of the propellant at both ends of the burner was replaced with some completely rigid material Then, neglecting interaction of waves reflected from the two different surfaces, the effective admittance of the boundary would be reduced in proportion to the amount of rigid surface area For this kind of experiment, Eq (10) would become

$$\alpha \simeq -[\beta/2 + 2(c/l)(A/A_0)Re(\rho c Y_b)] \qquad (15)$$

where A/A_0 is the fraction of the end-area covered by burning propellant If experiments were performed in which A/A_0 was systematically varied for a constant burner length, it can be seen that plotting measured values of α vs A/A_0 should result in a straight line with the slope equal to $-2(/cl)$ $Re(\rho c Y_b)$ and the intercept equal to $-\beta/2$

Figure 5 shows data obtained at NOTS for the forementioned propellant by the use of method III Values of A/A_0 greater than unity were obtained by adding a short length of perforated tubular propellant, having a very thin web, to the end-burning grain (Fig 6) If the length of this added propellant is short compared with the total length of the motor, the distortion of the wave motion should be small Data are shown for two different frequencies corresponding to two different motor lengths The data seem to conform to the predictions of Eq (14) very well for values of A/A_0 less than unity Reduced admittance values corresponding to these experiments are shown by two data points in Fig 7 For comparison, values obtained by the previous methods are also included The curve for method I represents admittances calculated by the use of minimum measured decay rates

It should be mentioned here that the use of more burning surface area than provided by the simple end-burning configuration has additional merits By providing more acoustic energy input, it permits the study of propellants that are naturally less prone to exhibit oscillatory combustion This has been demonstrated by recent work performed at the Bacchus Works of Hercules Powder Company[7] The data presented here confirm the usefulness of this method in obtaining estimates of propellant acoustic admittances, but

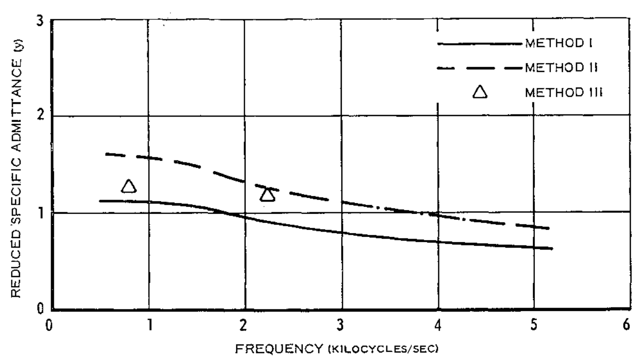

Fig 7 Comparison of admittance data for the three methods Method I data was calculated with minimum values of the damping constant

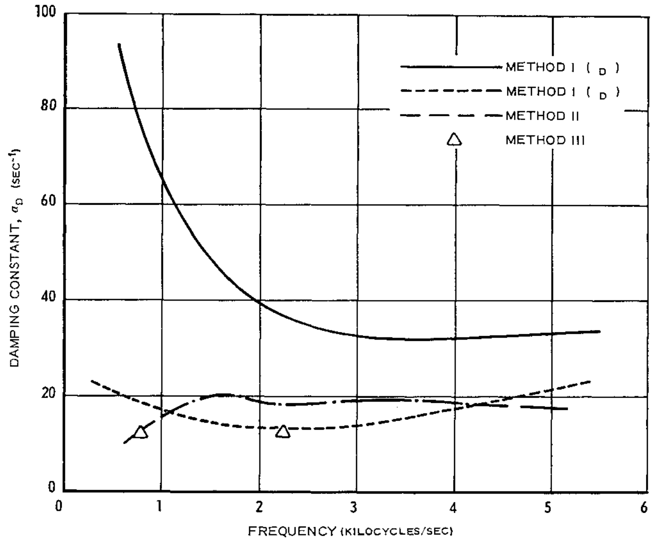

Fig 8 Comparison of the damping constants obtained by the three different methods

they suggest limitations on the extent that A/A_0 can be increased without appreciable error

Comparison of Damping Constants

Along with the comparison of the admittances, it is also of some interest to compare values of the damping constants for the three different methods Figure 8 shows curves representing average values for these data It is quite significant that methods II and III compare very well with the minimum values measured by method I This suggests that minimum values should be used in future applications of this technique

Conclusions

All three of the methods discussed here for employing the T-burner to measure the acoustic admittance of burning propellants have inherent weaknesses Nevertheless, the comparison of admittances obtained by the three substantially different methods should represent a good criterion for collective judgement of the T-burner approach in general It has been shown that if minimum values of the decay constant are used in method I the results for all three methods are consistent The agreement between admittances, as shown in Fig 7, is well within the experimental accuracy On the basis of this comparison, the T-burner approach seems a valid technique for determining the effective specific acoustic admittance of burning solid propellants

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